Targeted matrix completion

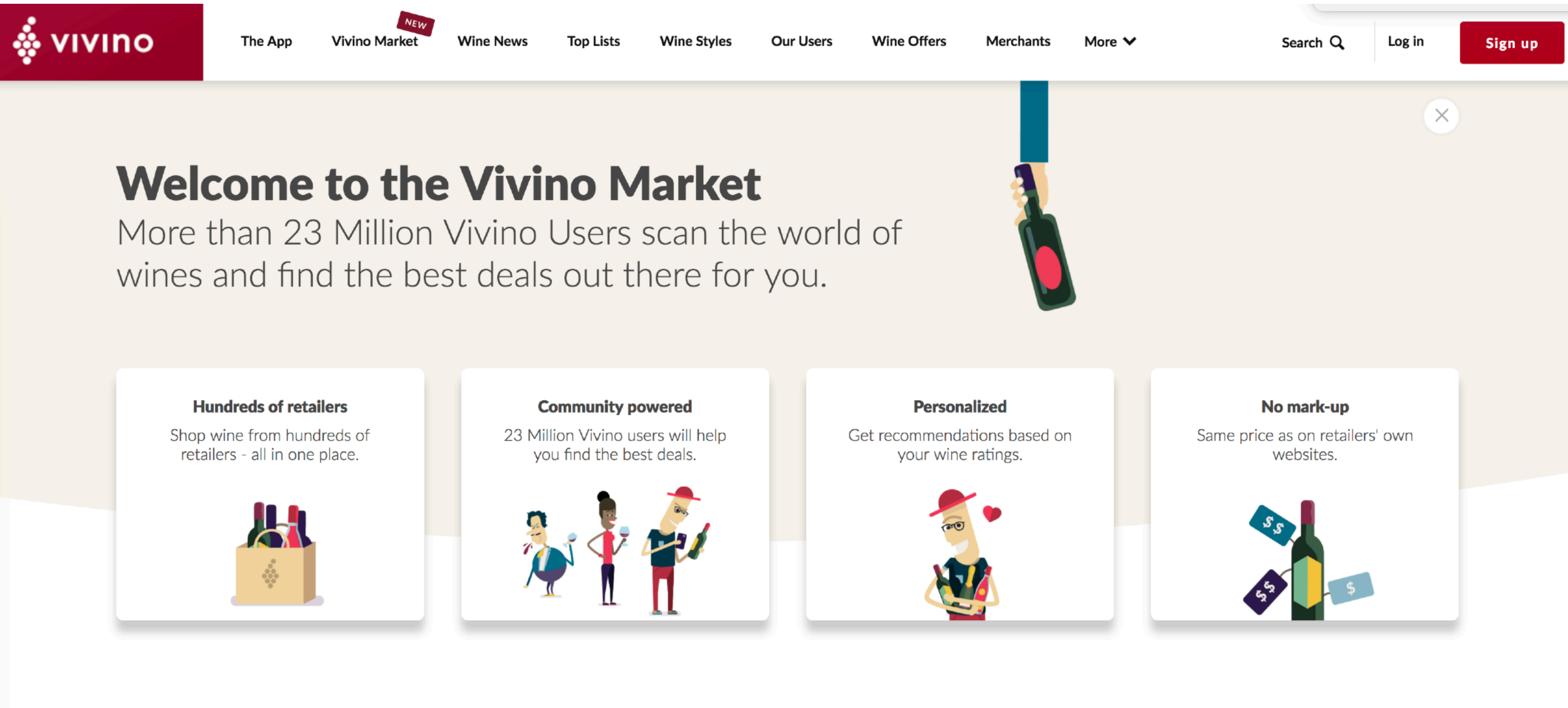
Natali Ruchansky

Mark Crovella and Evimaria Terzi

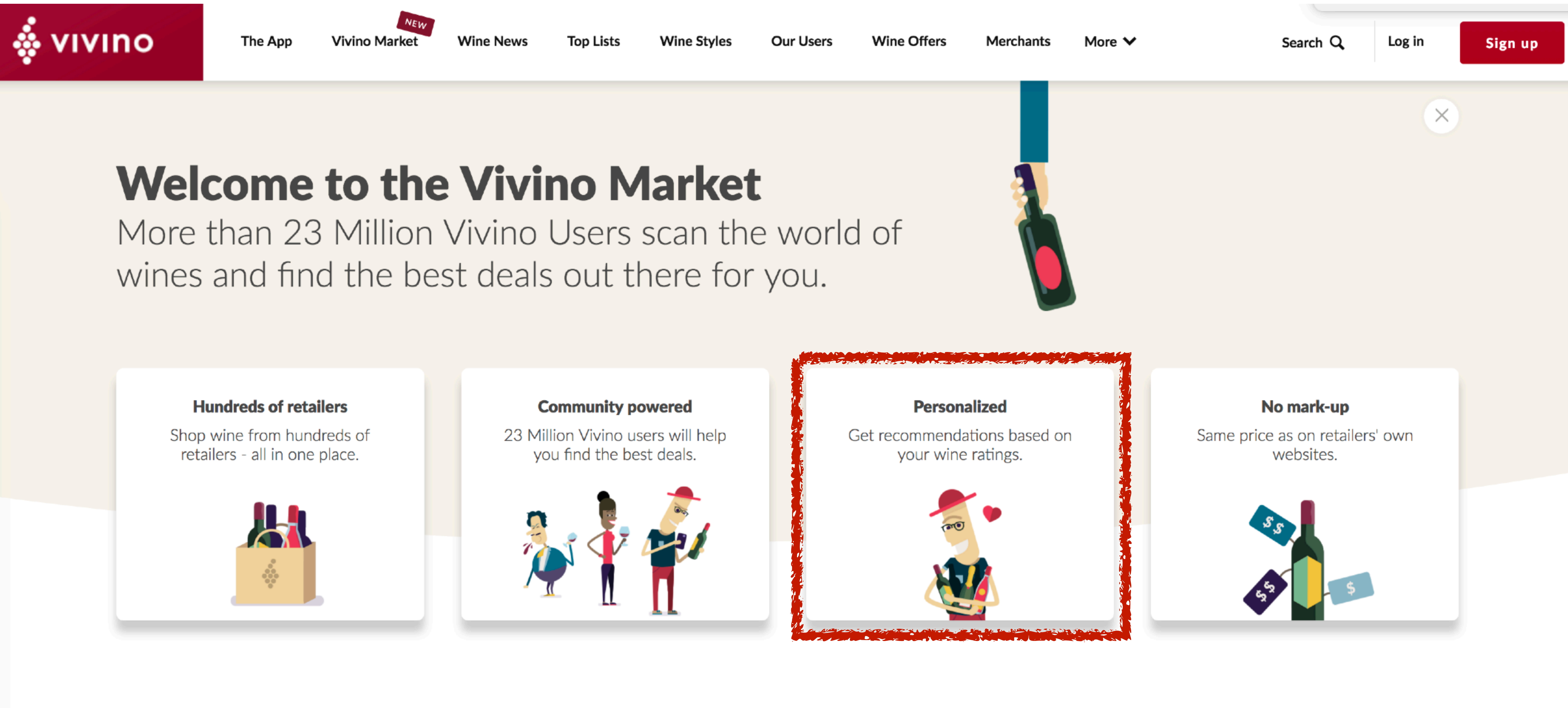
Planning a dinner and looking for a wine that you **haven't tried** before and you **will enjoy**.

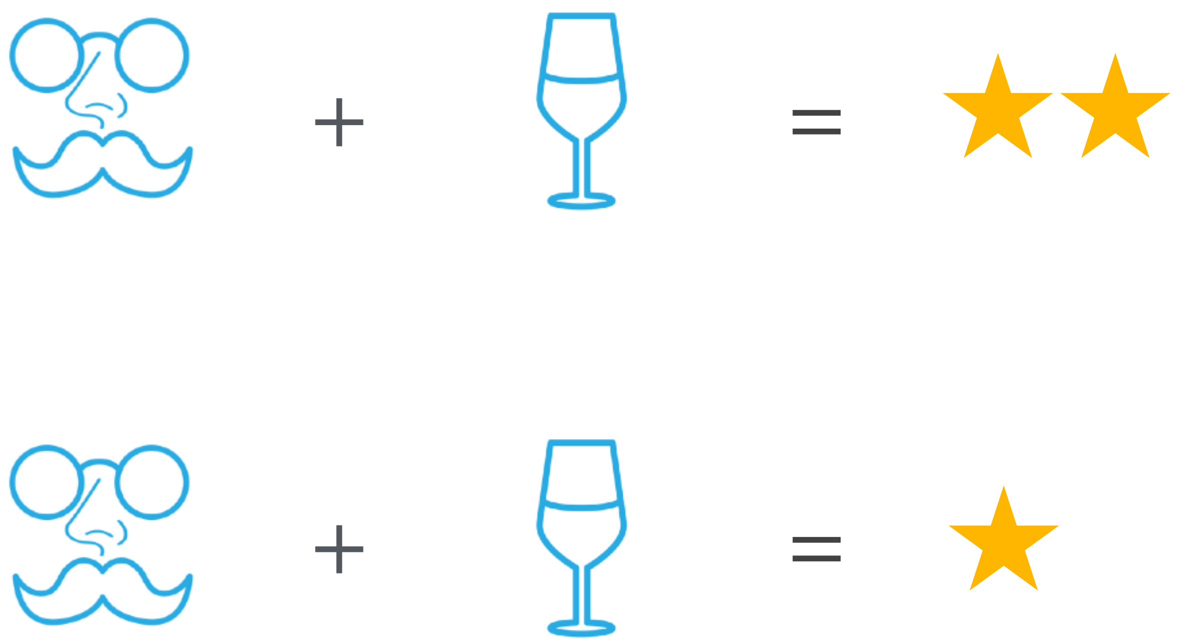
Skeptical of the liquor store owner's monetary incentive.

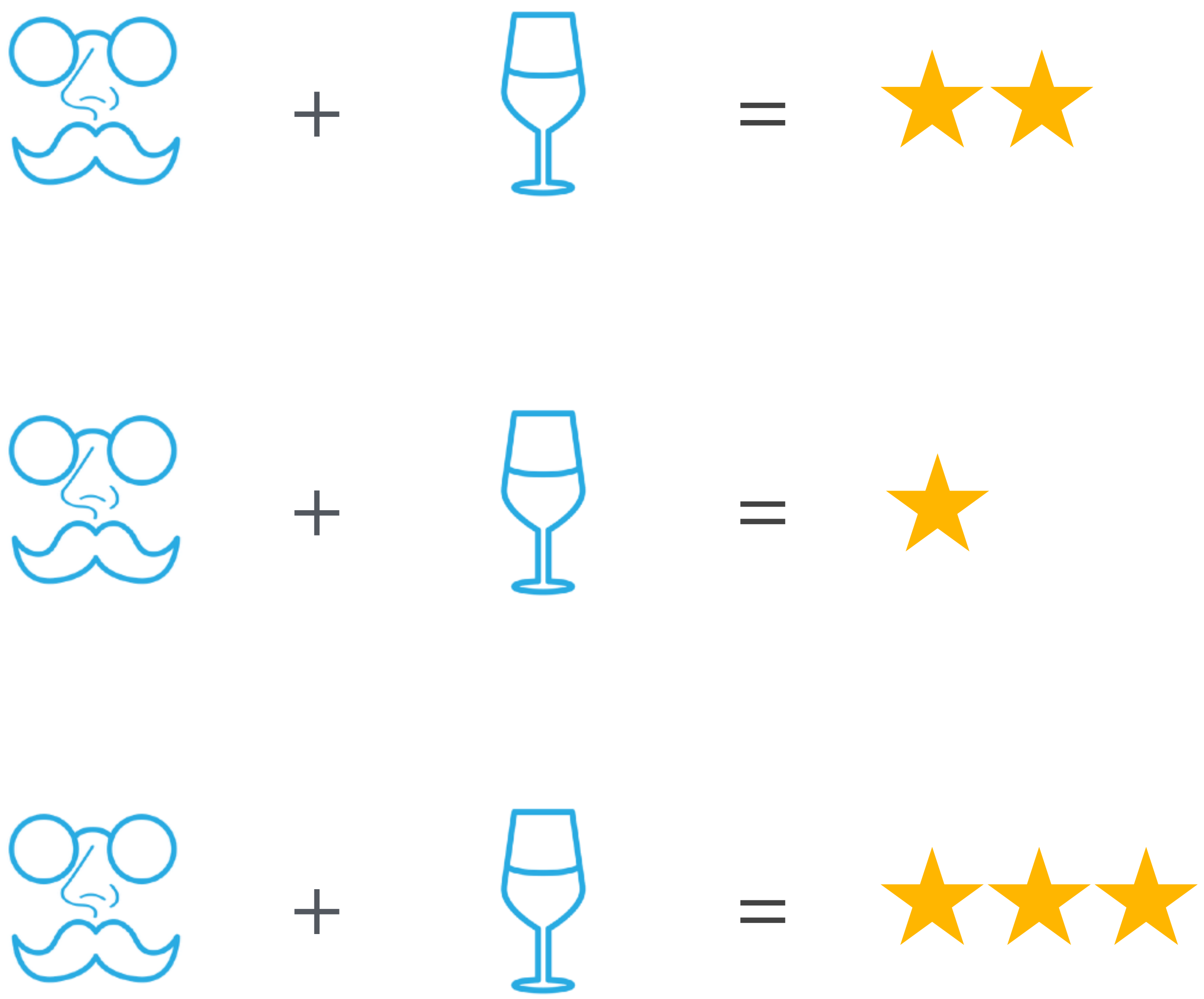
Luckily, there is Vivino.



Luckily, there is Vivino.



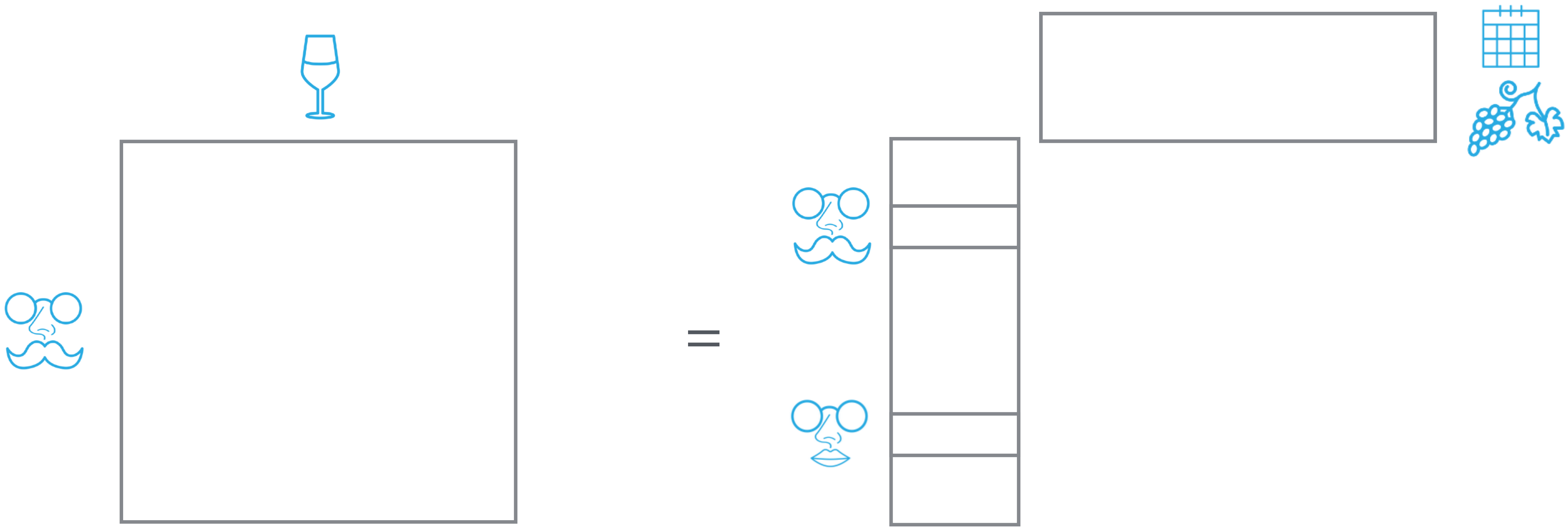




Matrix completion

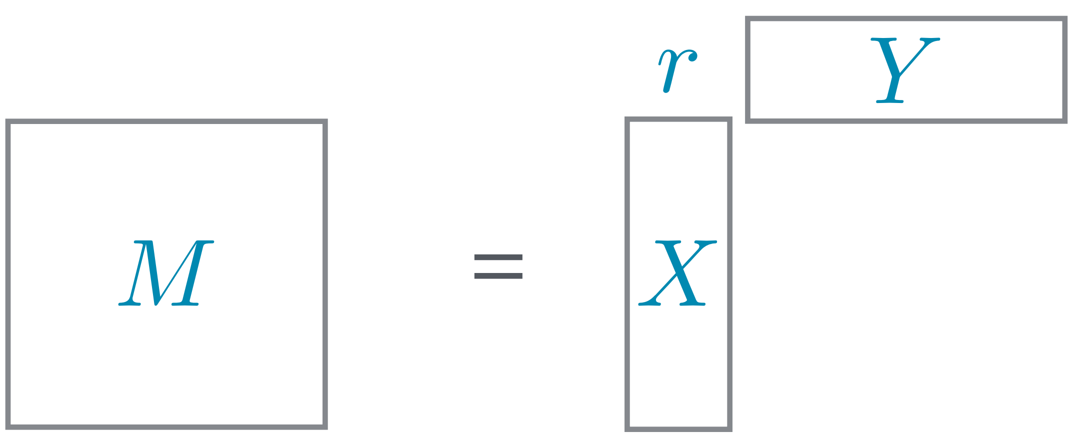
given a partially observed matrix, estimate the missing values

Data has low rank



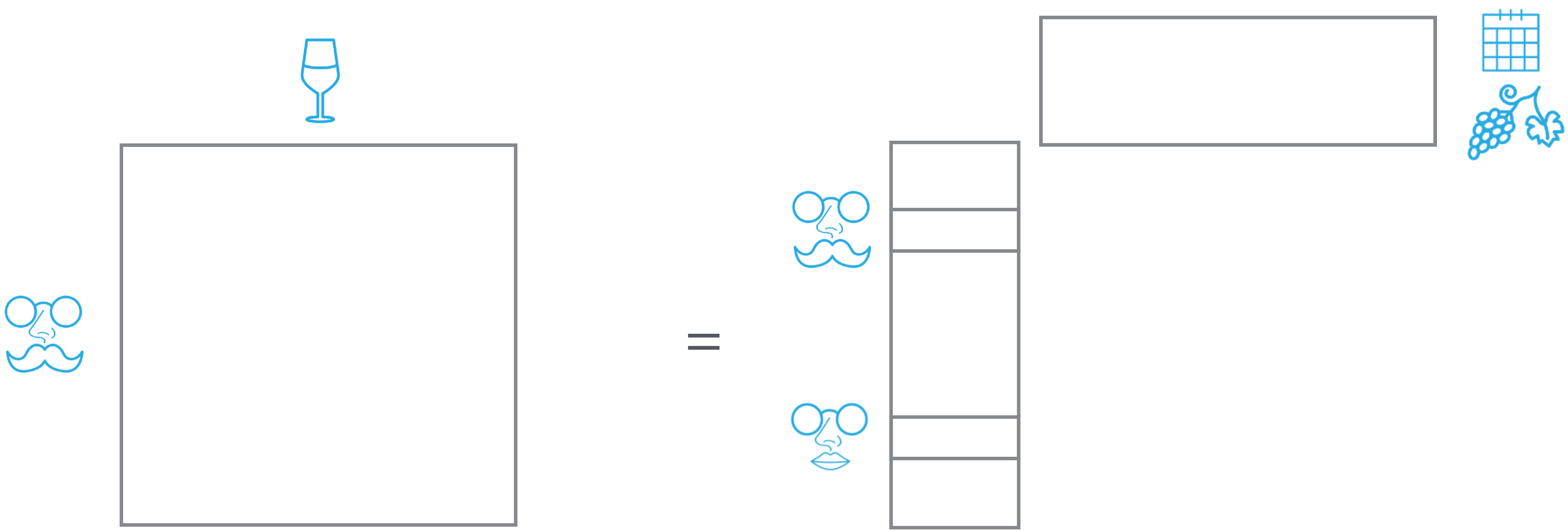
Low-rank matrix completion

Given a matrix M_Ω observed only at locations Ω , find a low-rank estimate that matches on Ω.

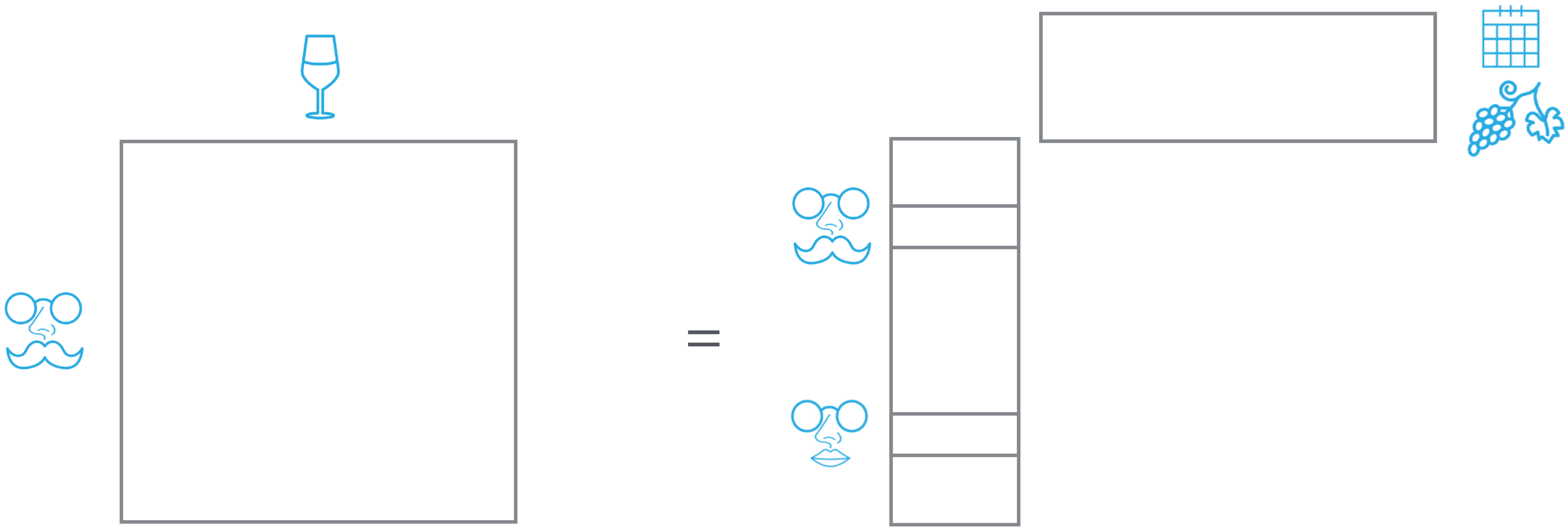


$$\min \|\hat{M} - XY\|$$
$$\text{subject to } \hat{M}_\Omega = M_\Omega$$

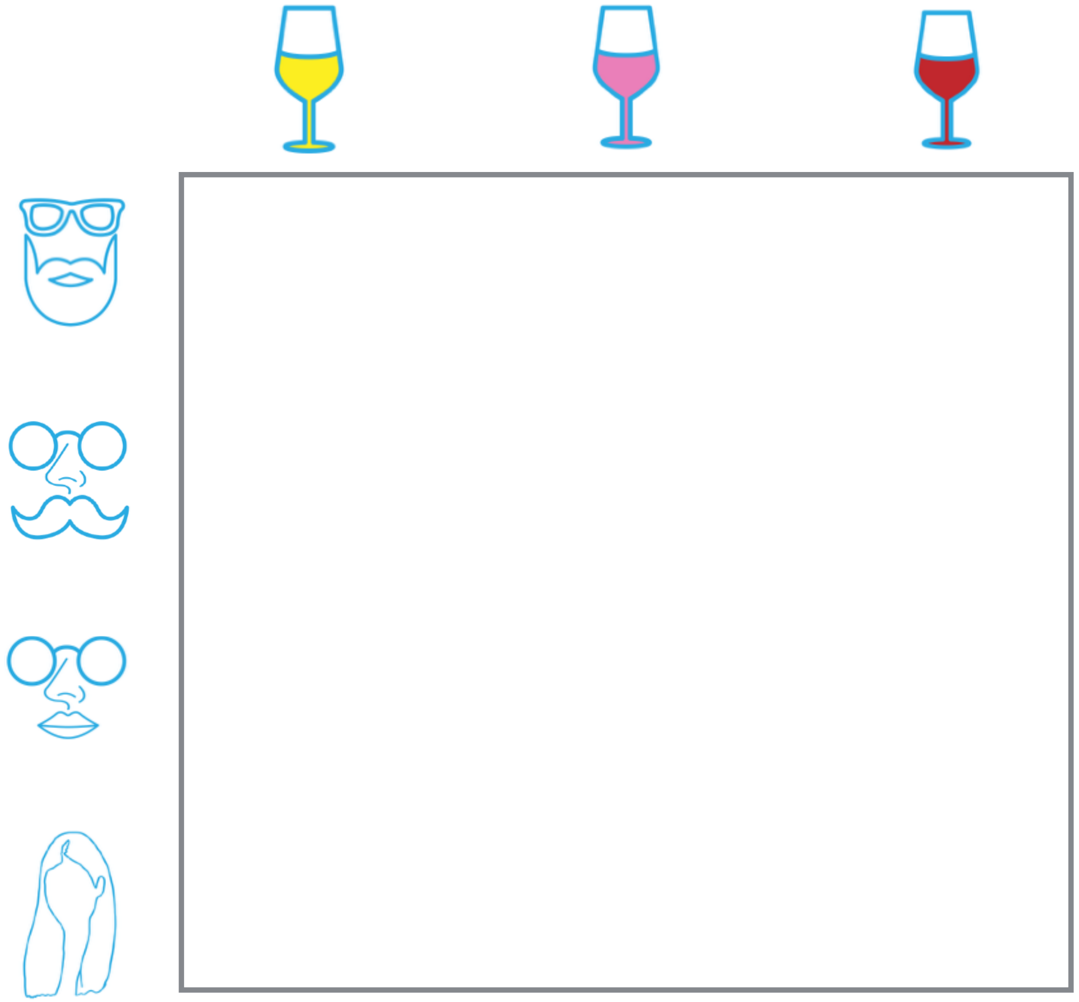
Data has low rank

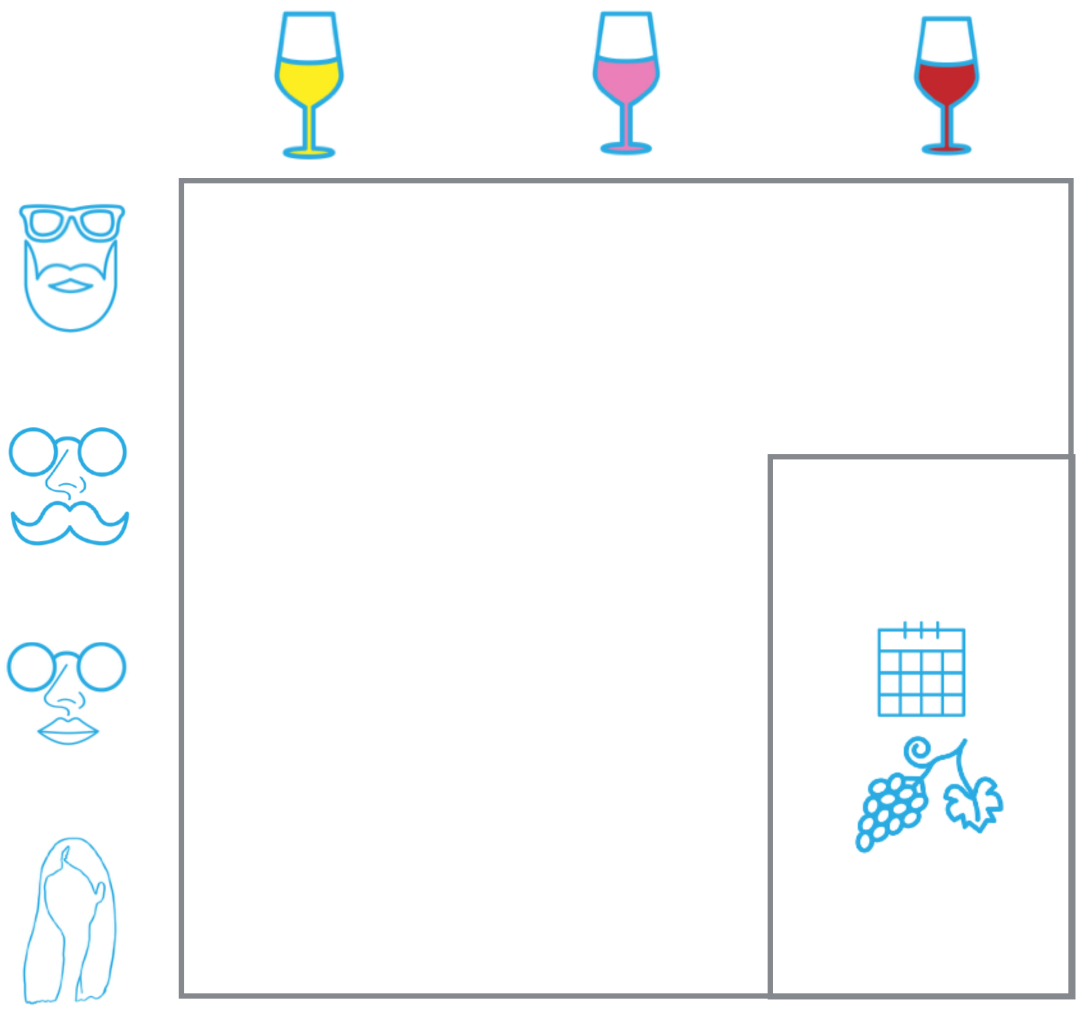


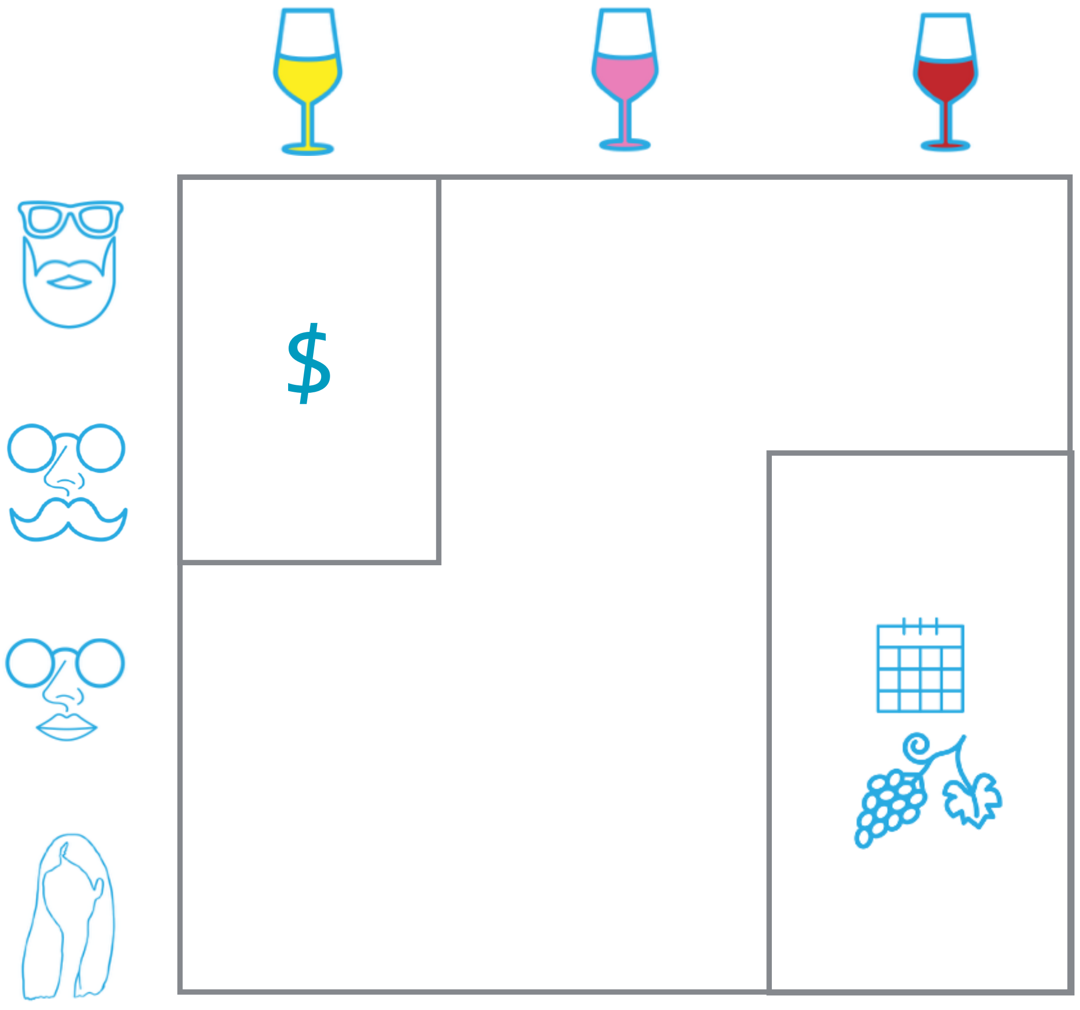
Data has low rank



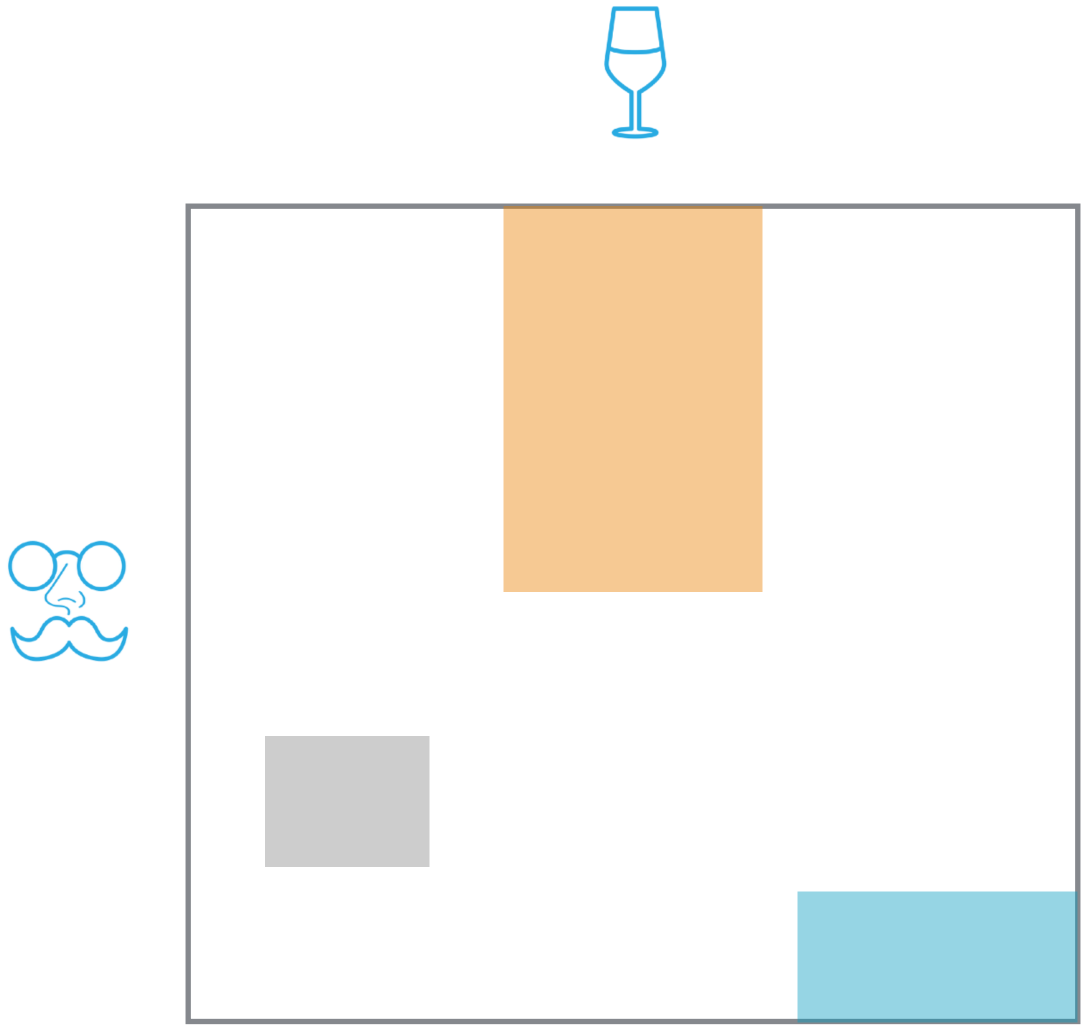
real preferences are more complex



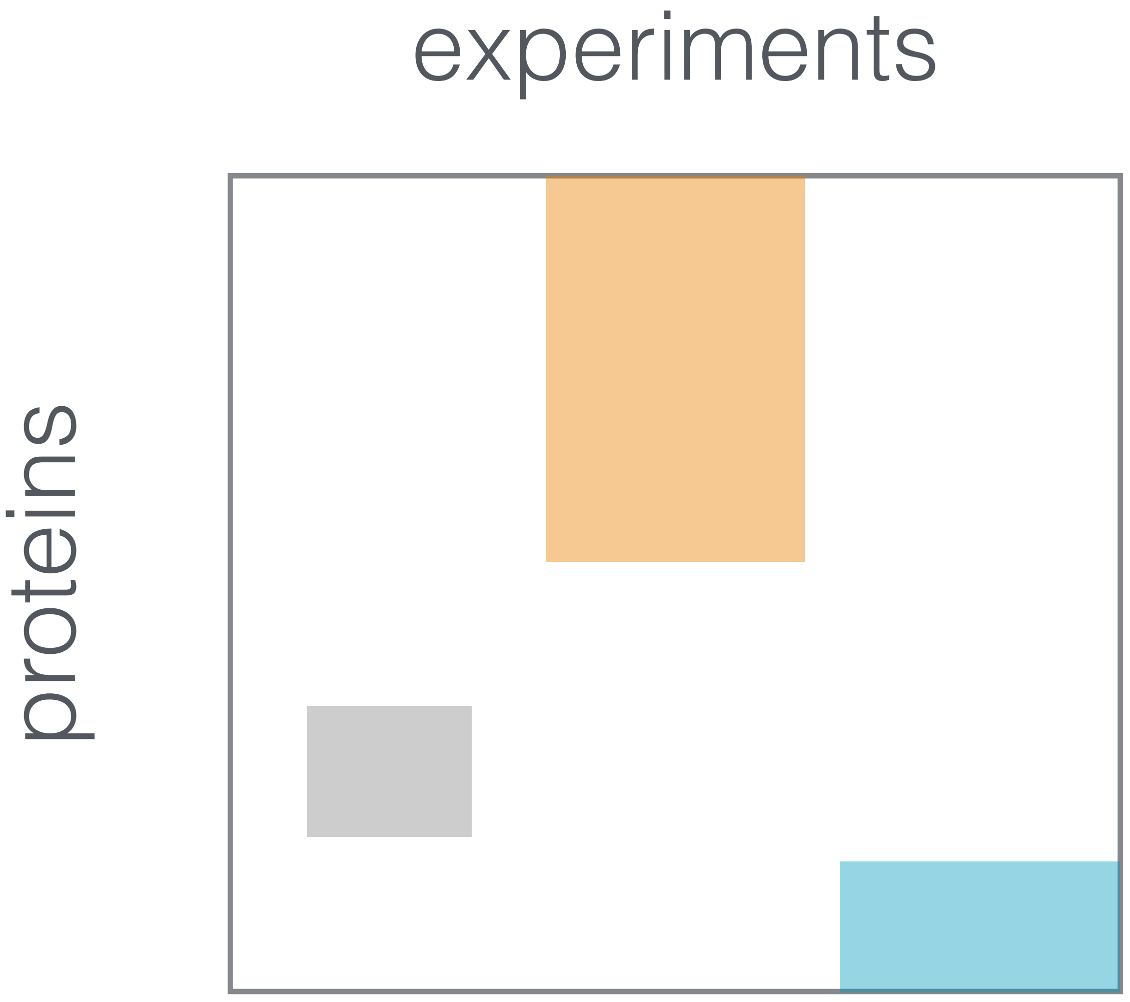




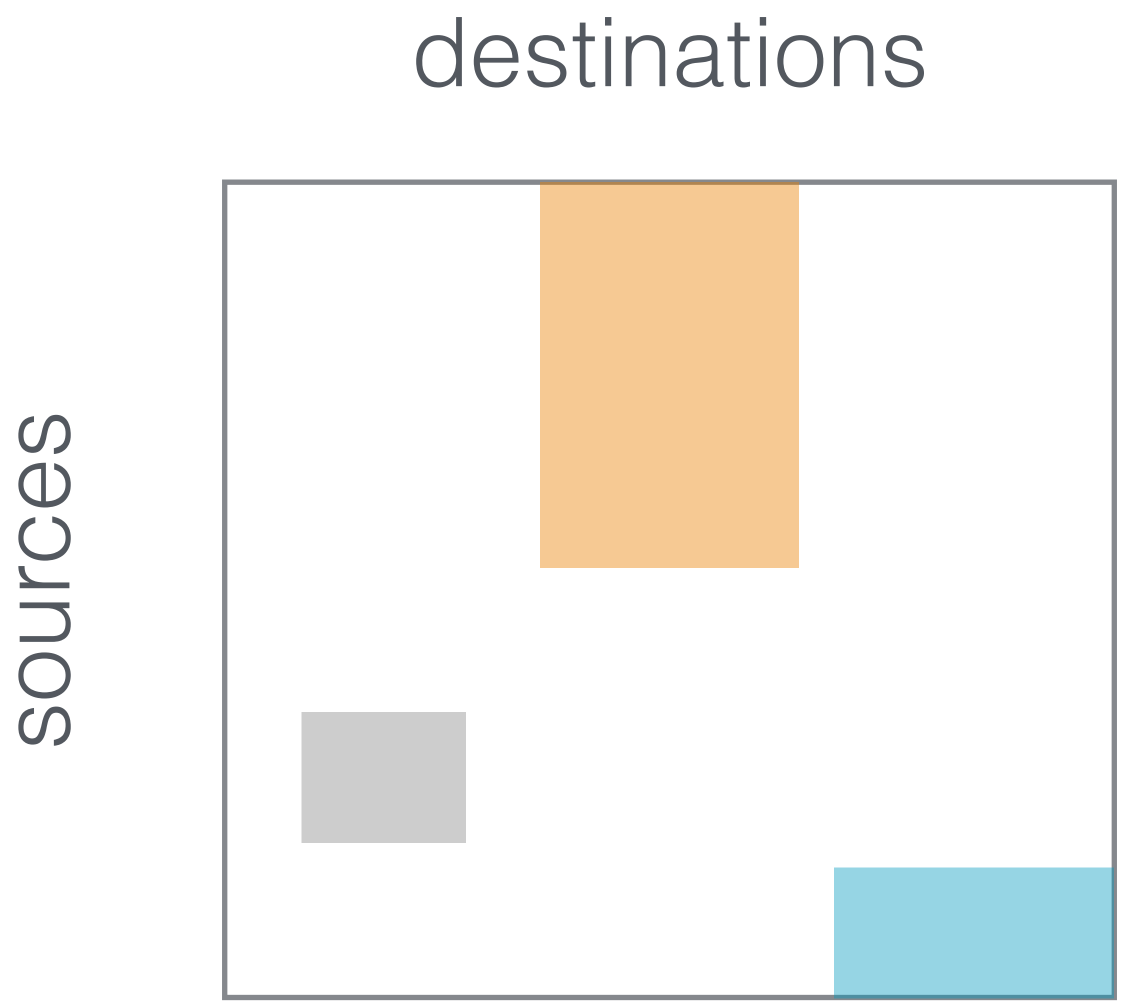
Data has low-rank submatrices



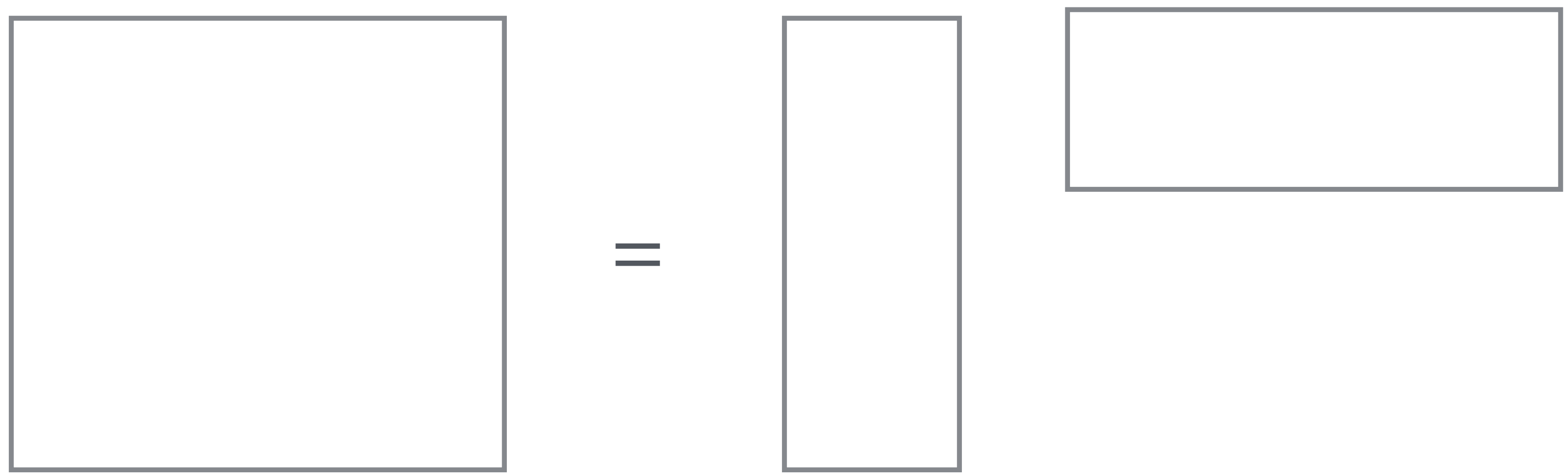
Data has low-rank submatrices



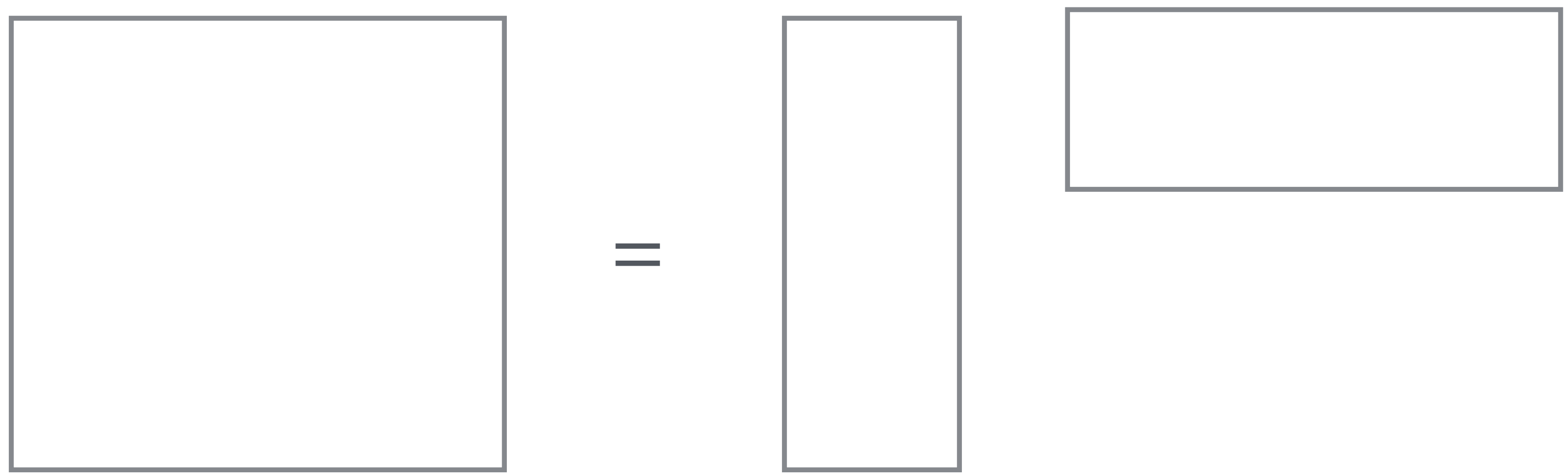
Data has low-rank submatrices



Traditional matrix completion algorithms are designed for the global low-rank assumption



Traditional matrix completion algorithms are designed for the global low-rank assumption

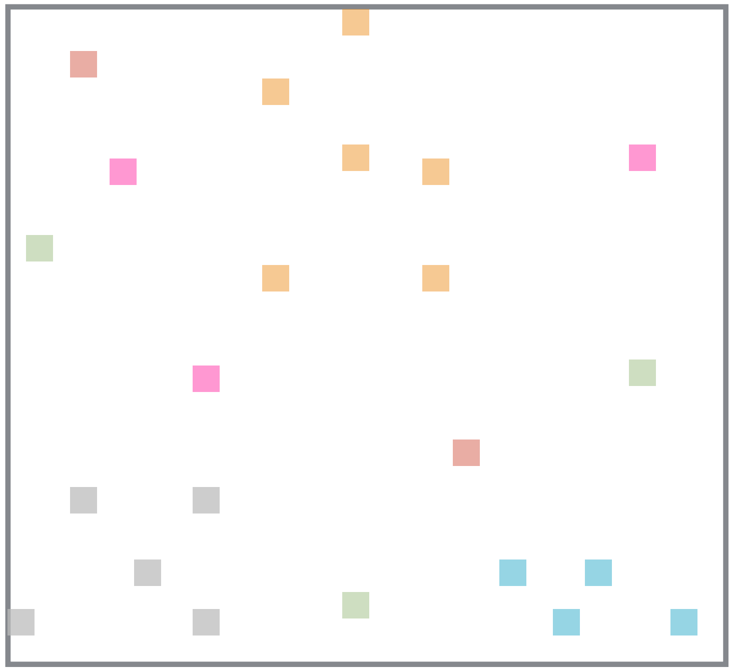


and do not produce an accurate estimate when the input is composed of low-rank submatrices

How can we accurately complete
a partially-observed matrix of this form?

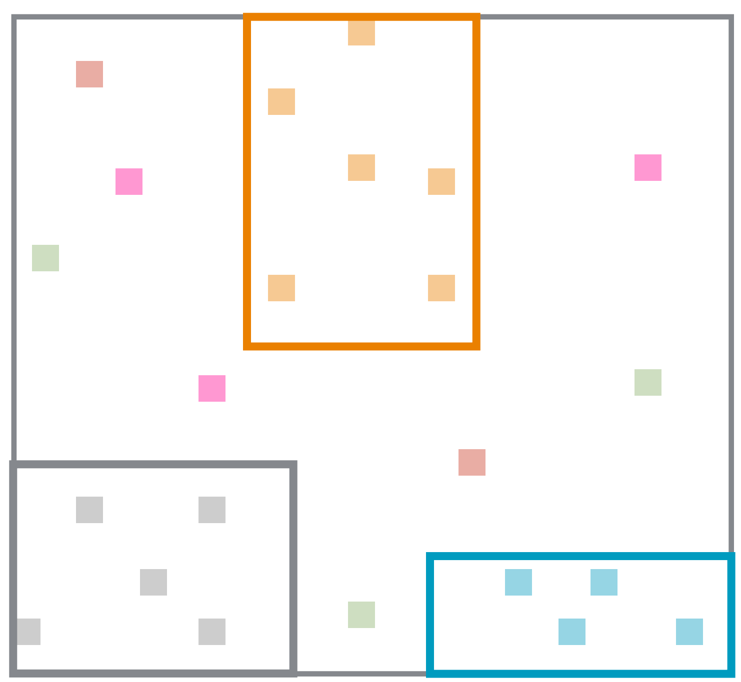
Targeted matrix completion

1. **Find** low-rank submatrices



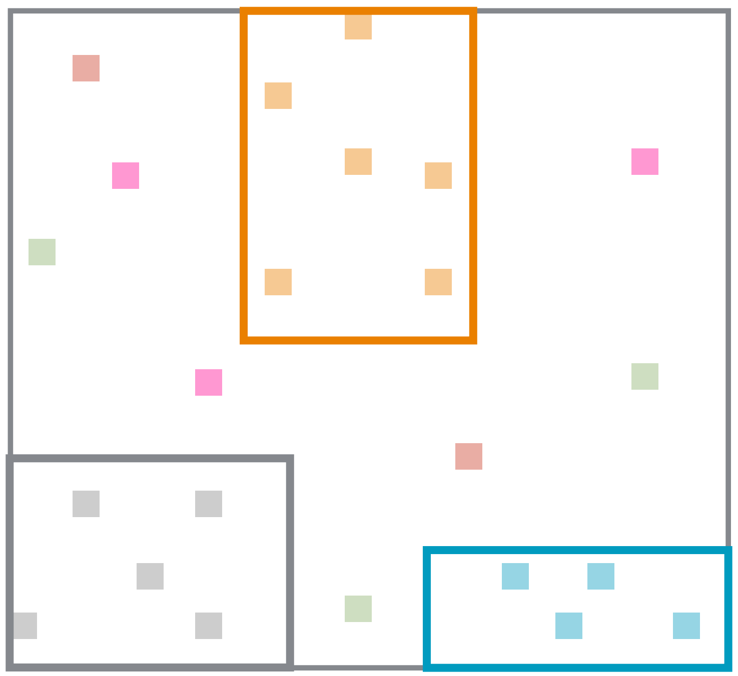
Targeted matrix completion

1. **Find** low-rank submatrices



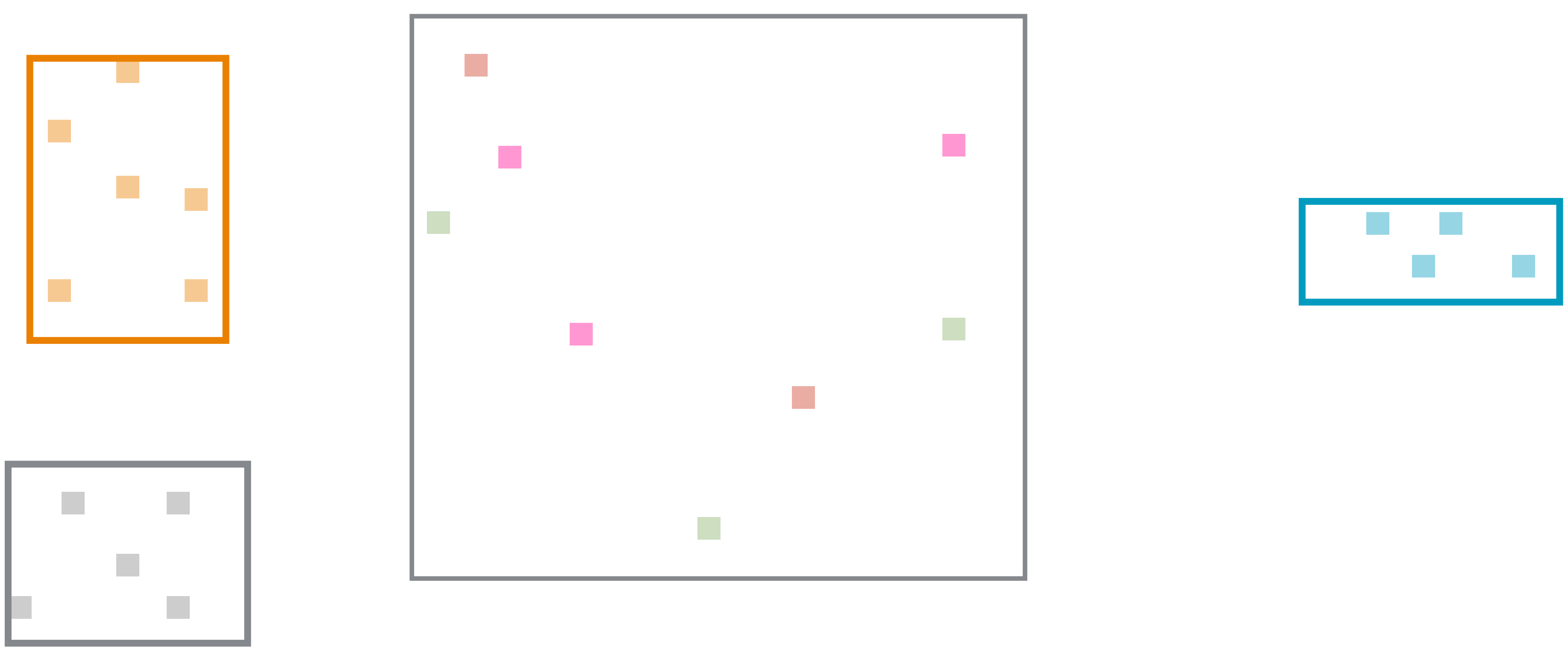
Targeted matrix completion

1. Find low-rank submatrices
2. **Separate** them



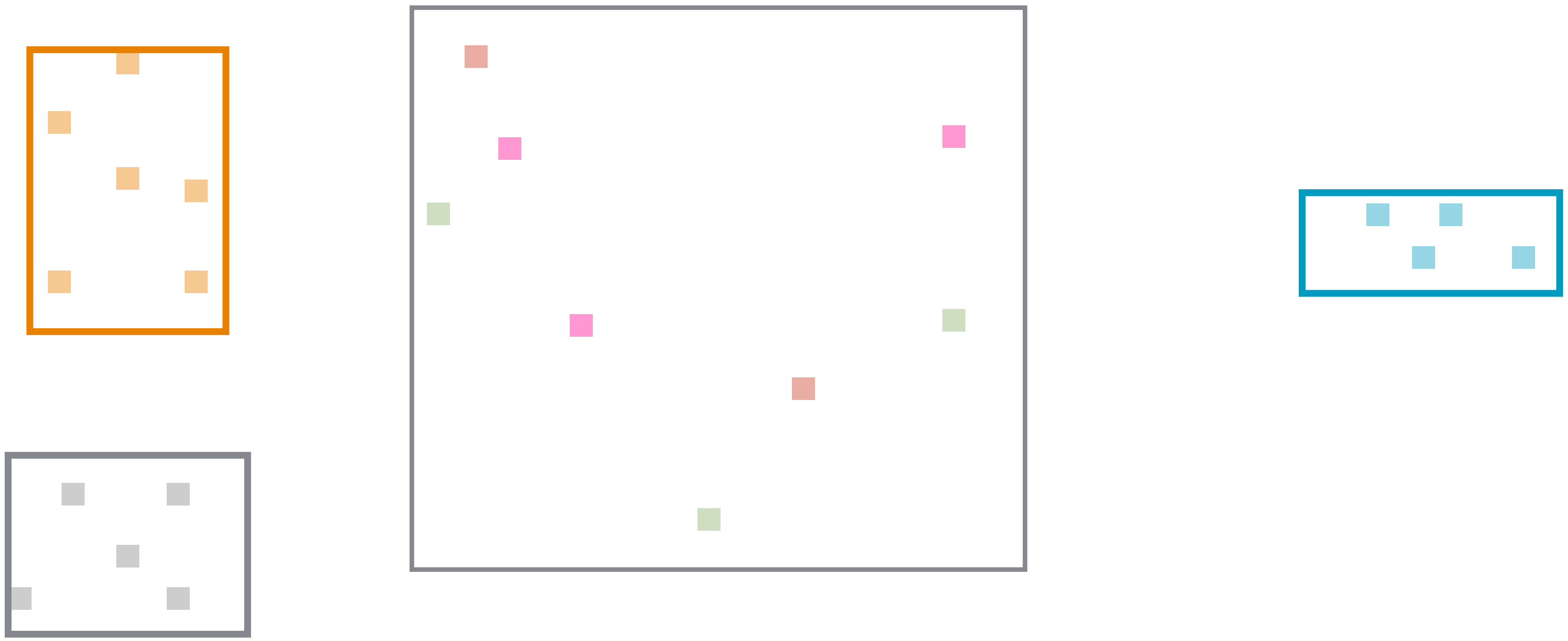
Targeted matrix completion

1. Find low-rank submatrices
2. **Separate** them



Targeted matrix completion

1. Find low-rank submatrices
2. Separate them
3. **Complete** each

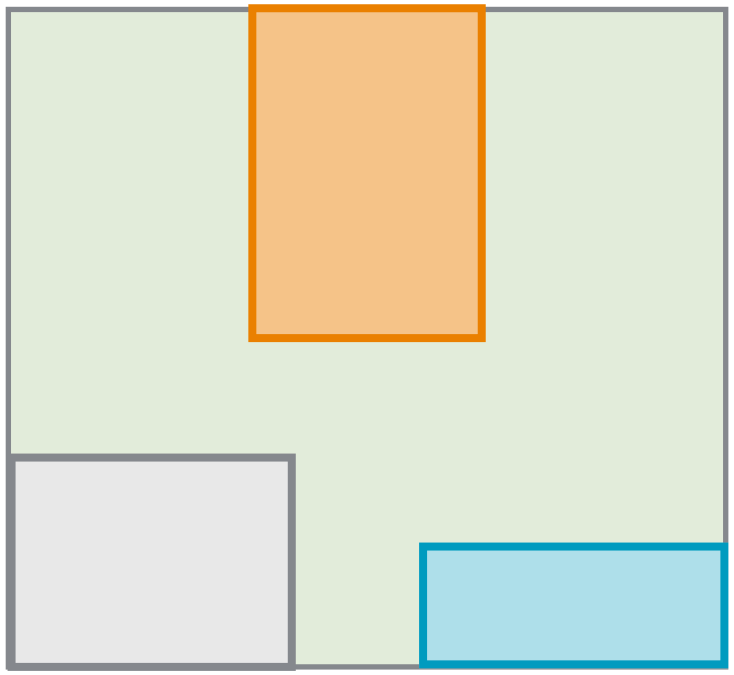


Targeted matrix completion

1. Find low-rank submatrices
2. Separate them
3. **Complete** each

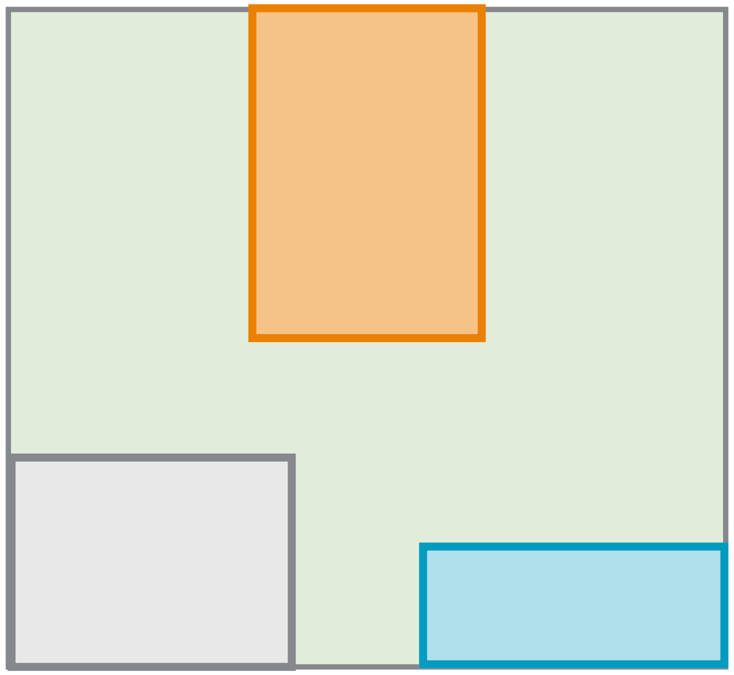
Targeted matrix completion

1. Find low-rank submatrices
2. Separate them
3. Complete each
4. **Combine** the completions



Targeted matrix completion

1. Find low-rank submatrices
2. Separate them
3. Complete each
4. Combine the completions



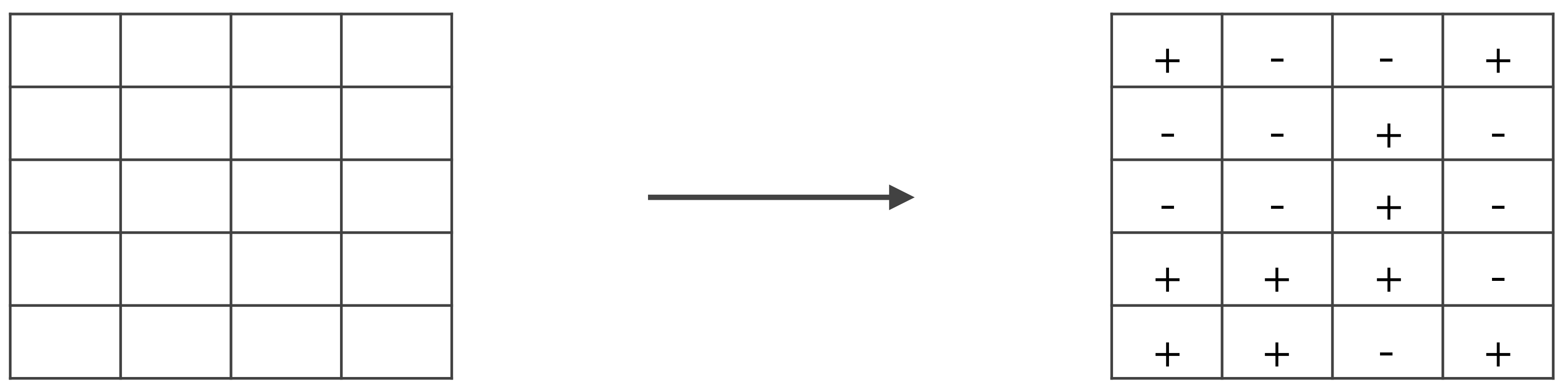
How do we **find** low-rank submatrices?

How do we **find** low-rank submatrices?

Interesting even for the fully-known case…

Previous work

[Rangan 2012] showed that the ± pattern can be used to find a low-rank submatrix in a **fully known** matrix.



Works for a single **large** submatrix with rank **<5** if the data is from a standard **Gaussian**.

Previous work

[Rangan 2012] showed that the ± pattern can be used to find a low-rank submatrix in a fully known matrix.

Other related works ([Vidal 2014]) similarly require,

- fully-known matrix
- specific distribution
- large submatrix
- fully-column submatrix

Or are expensive.

Previous work

[Rangan 2012] showed that the ± pattern can be used to find a low-rank submatrix in a fully known matrix.

Other related works ([Vidal 2014]) similarly require,

- fully-known matrix
- specific distribution
- large submatrix
- fully-column submatrix

Or are expensive.

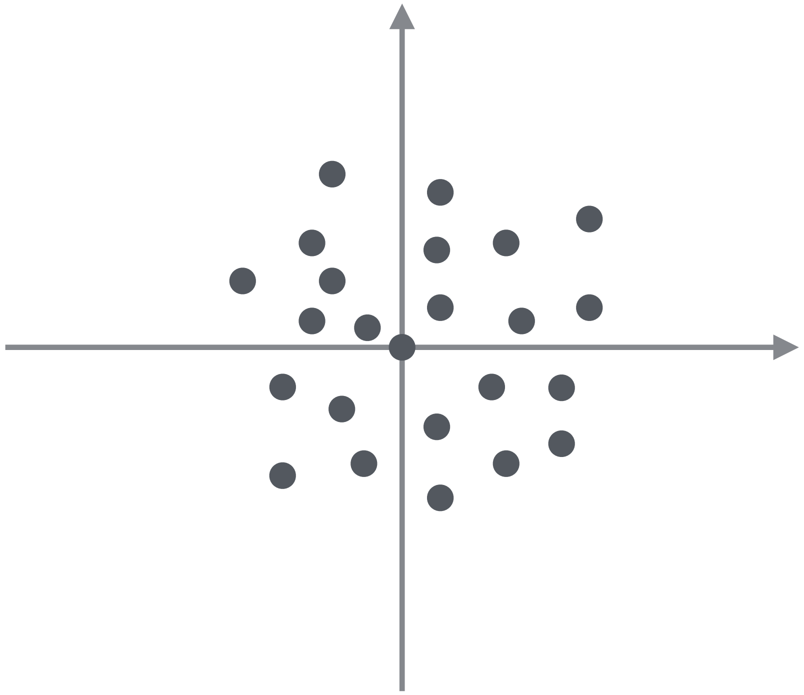
Our approach is more **general** and more **efficient**

Special case

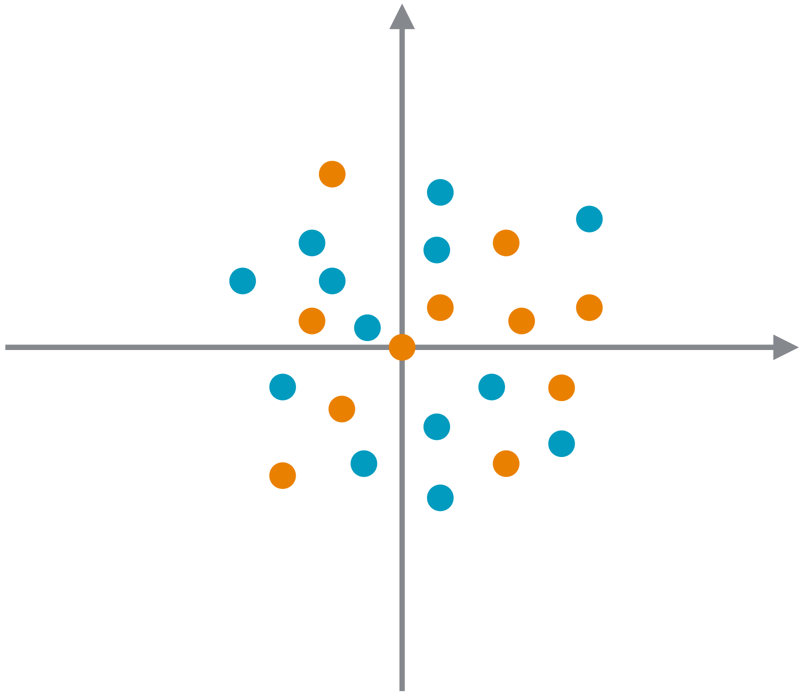
Special case

Fully known matrix with one low-rank submatrix spanning all columns.

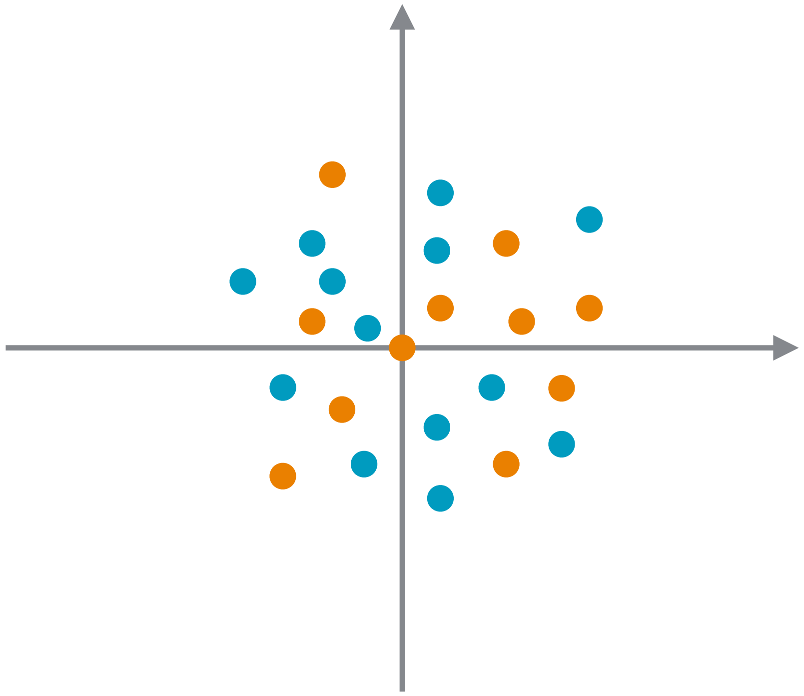
Special case



Special case

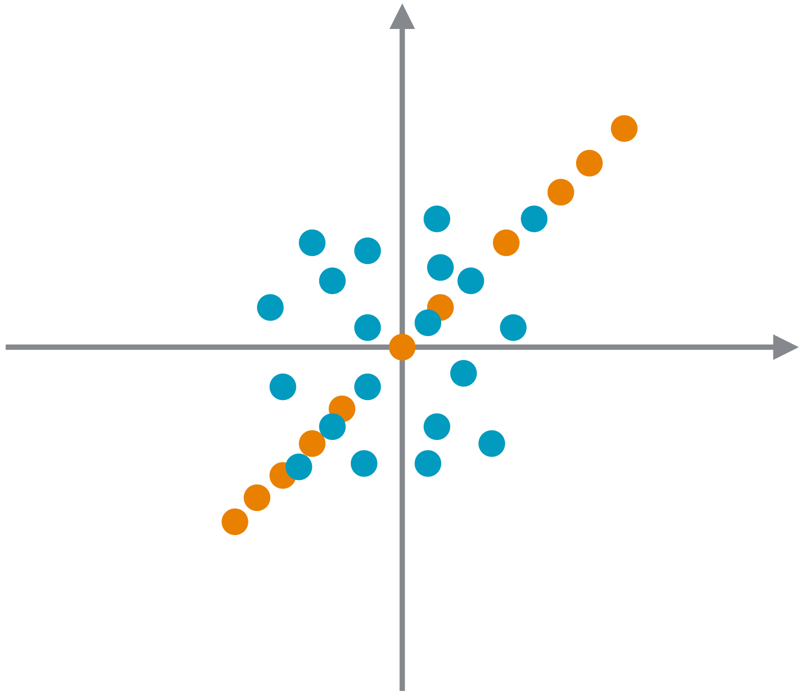


Special case



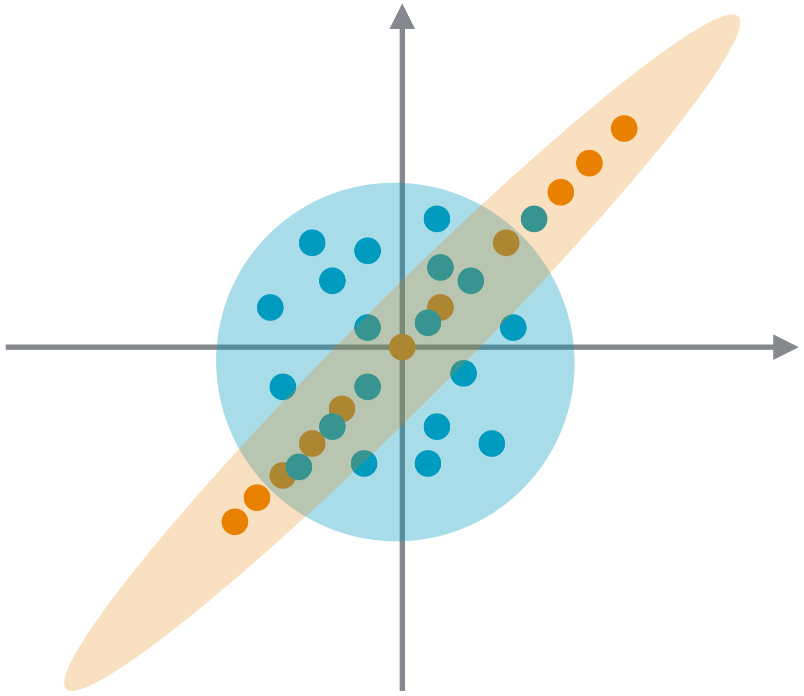
Special case

if the **low-rank submatrix** is larger in magnitude



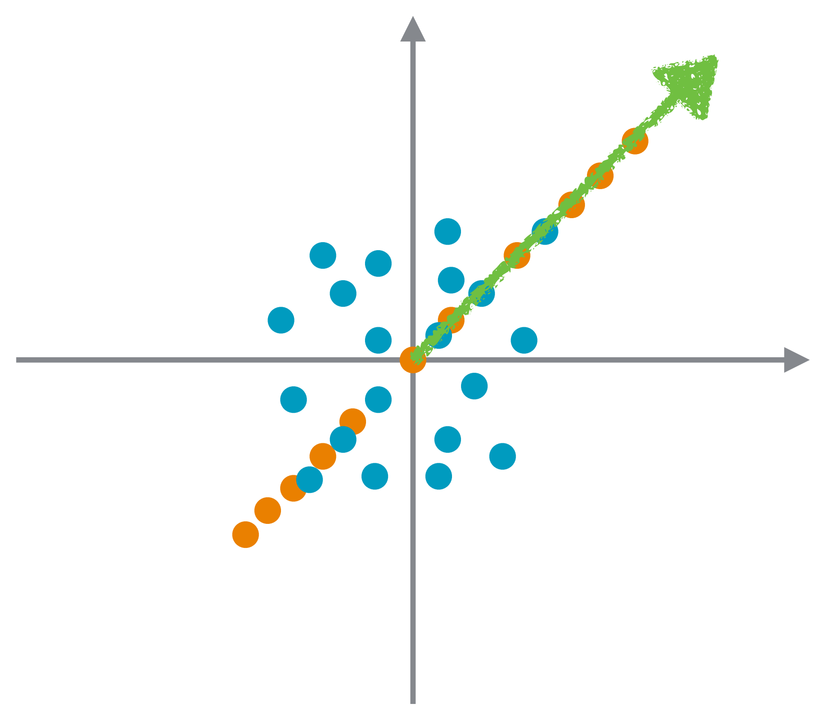
Special case

if the **low-rank submatrix** is larger in magnitude



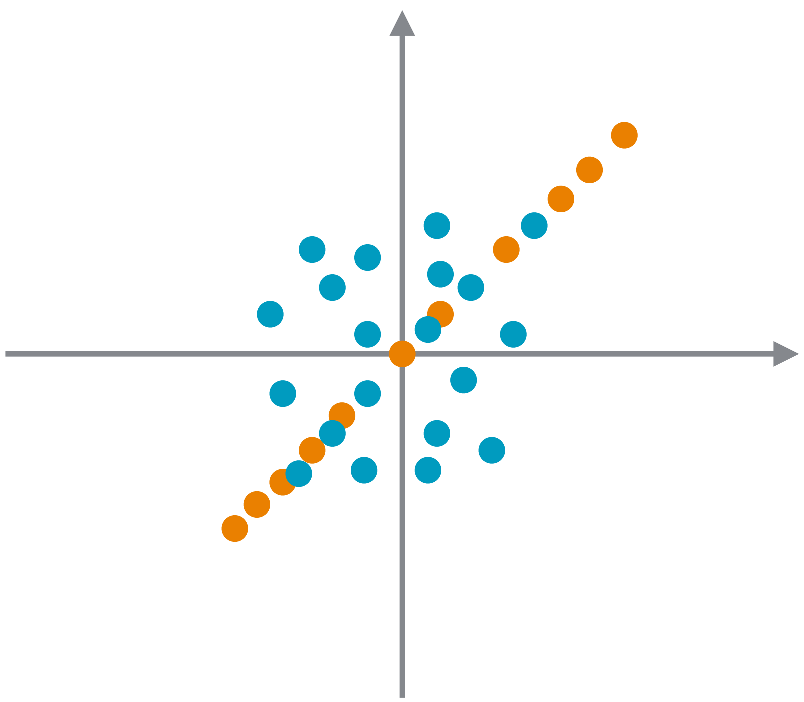
Special case

if the **low-rank submatrix** is larger in magnitude
then the first **singular vector** will orient towards it



Special case

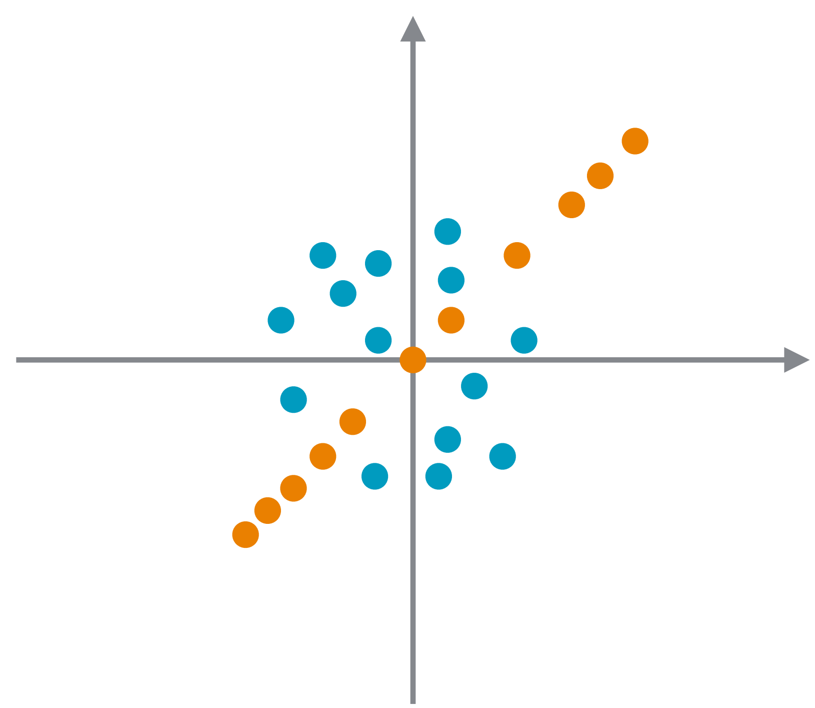
if the **low-rank submatrix** is larger in magnitude then the first **singular vector** will orient towards it



if the **low-rank submatrix** is also well separate

Special case

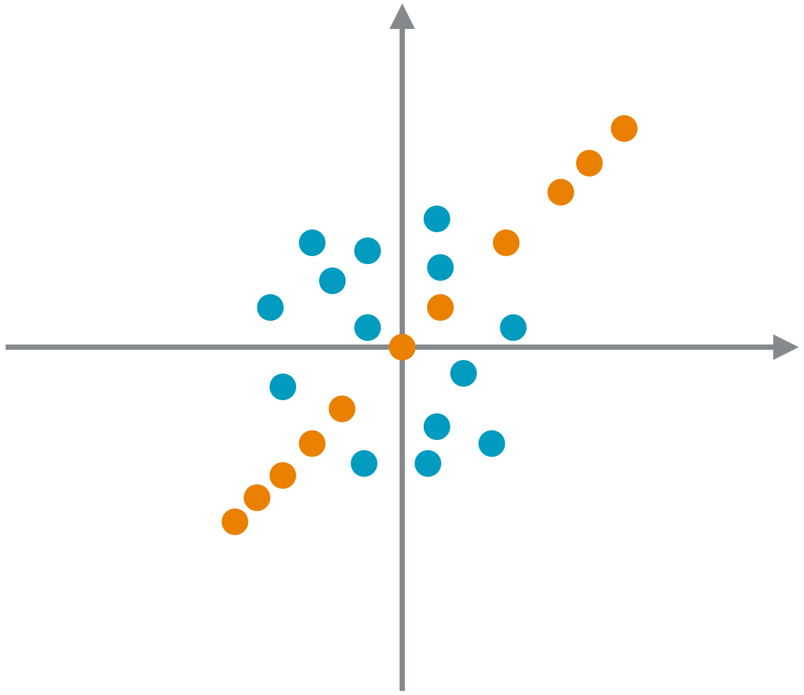
if the **low-rank submatrix** is larger in magnitude then the first **singular vector** will orient towards it



if the **low-rank submatrix** is also well separate

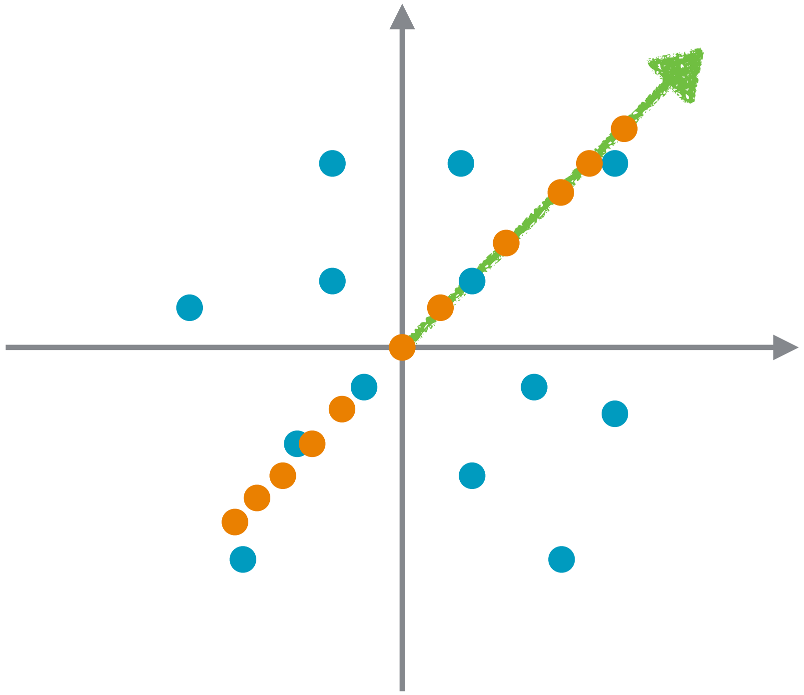
Special case

if the **low-rank submatrix** is larger in magnitude
then the first **singular vector** will orient towards it



if the **low-rank submatrix** is also well separate
then the first **singular vector** can be used to find it

Special case



Special case

Special case

Project and partition

Simply two-part algorithm:

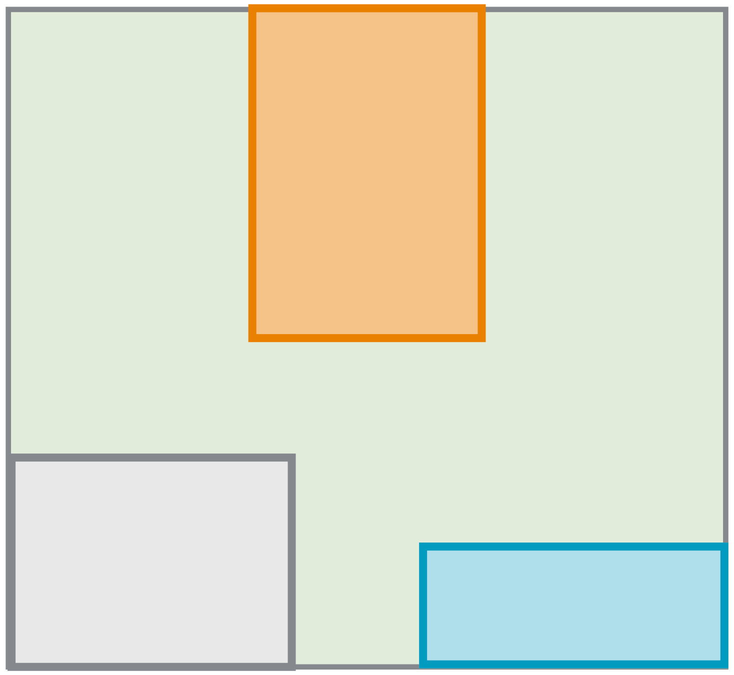
Project: each row and columns onto the first singular vector

Partition: projections to separate low-rank submatrix

Apply iteratively for multiple.

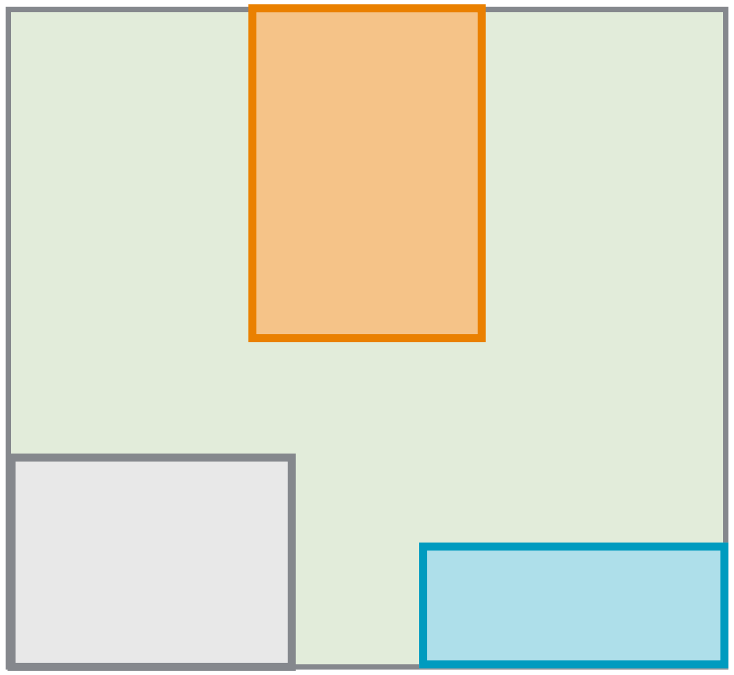
Targeted matrix completion

1. Find low-rank submatrices
2. Separate them
3. Complete each
4. Combine the completions

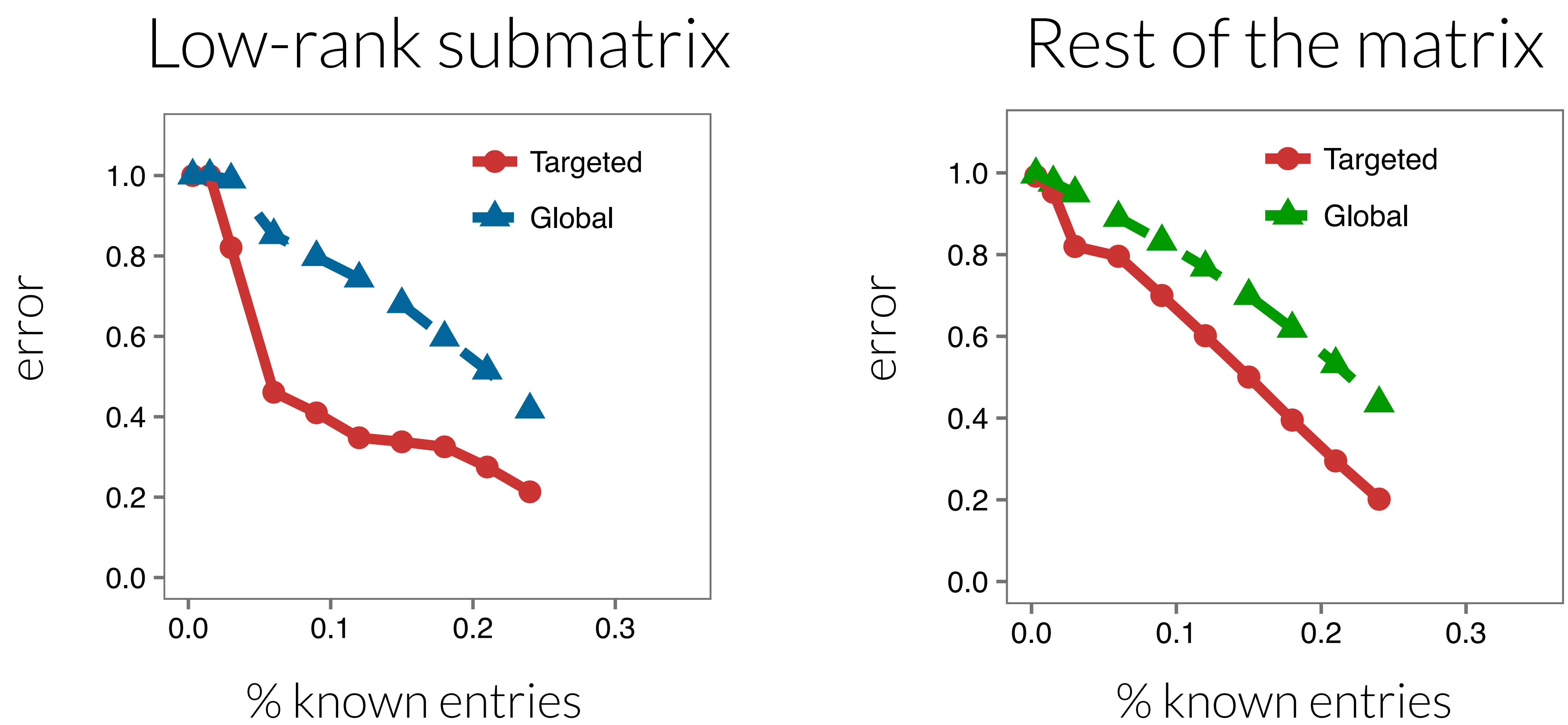


Targeted matrix completion

1. Find low-rank submatrices
2. Separate them
3. Complete each
4. Combine the completions



Internet Traffic



source-destination traffic volume matrix.

The paper shows…

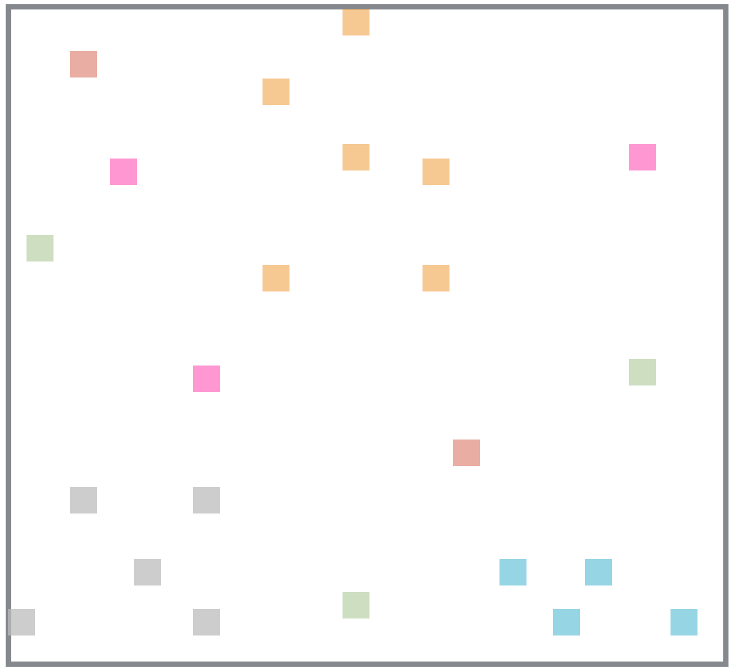
The Project&Partition approach extends to **partially-observed** matrices that have **multiple** low-rank submatrices of arbitrary shape.

Conditions under which the algorithm (SVP) identifies low-rank submatrices.

SVP is more accurate than existing approaches.

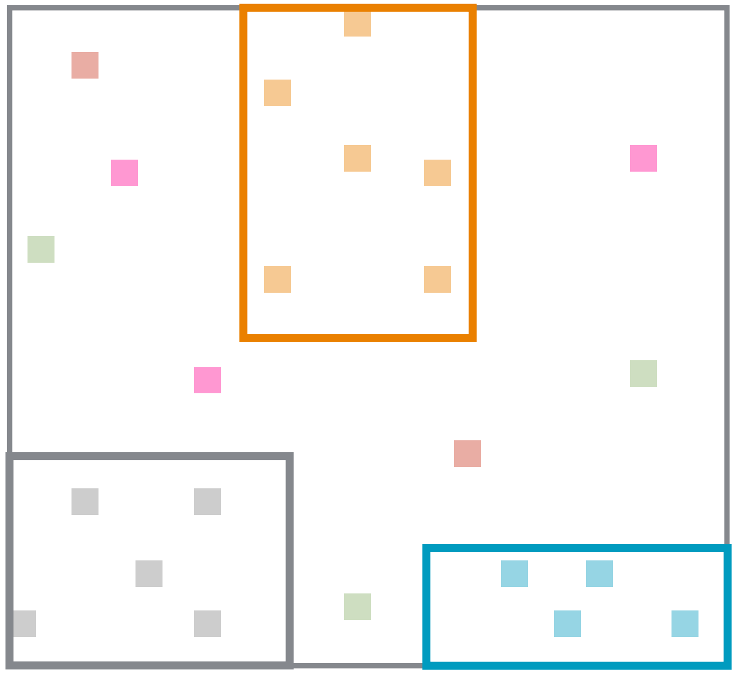
Targeted matrix completion leads to a more **accurate** and **efficient** completion on **real-world** and synthetic data.

Targeted matrix completion



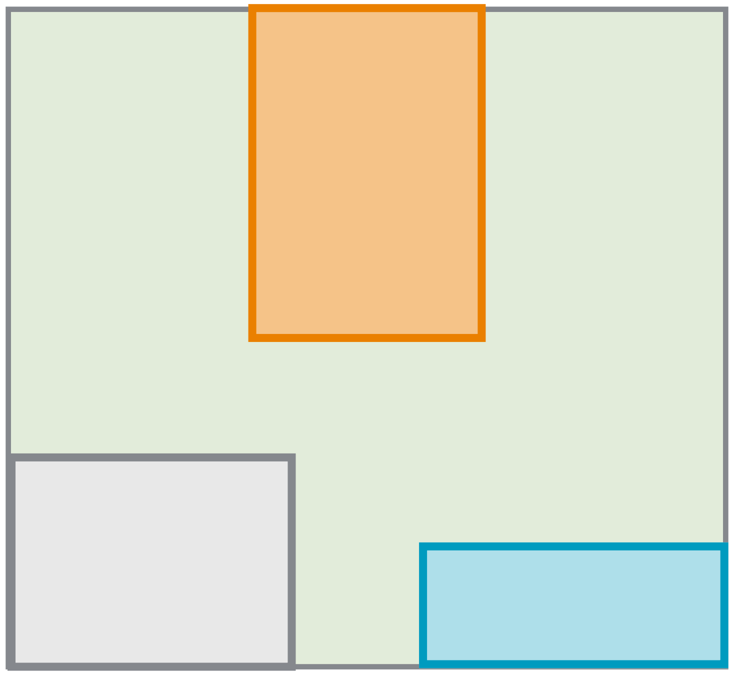
given a partially observed matrix
with low-rank submatrices

Targeted matrix completion



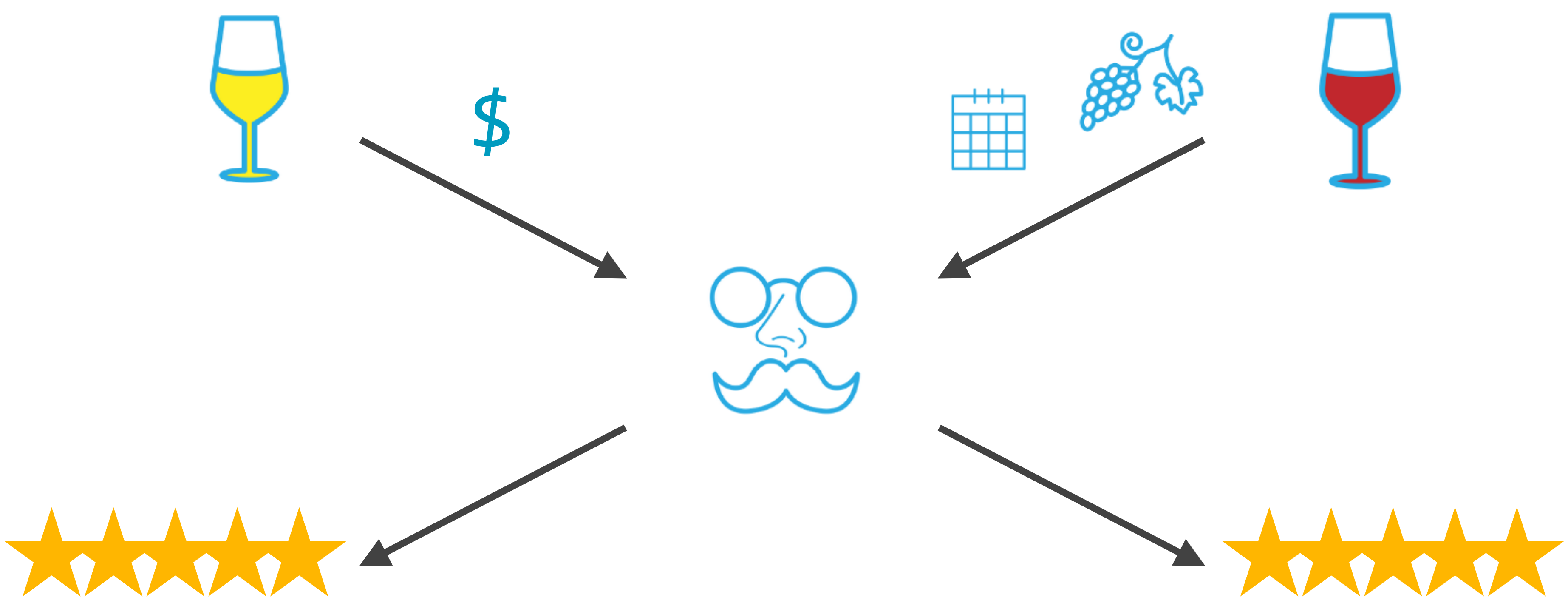
find the low-rank submatrices

Targeted matrix completion



(more) accurately complete the matrix

Targeted matrix completion



*thanks to AutoDraw for illustration support

Thanks for listening.